Emergent Photons and Gravitons: The Problem of Vacuum Structure *†

J. D. BJORKEN
SLAC National Accelerator Laboratory
2575 Sand Hill Road
Menlo Park, CA 94025/Pacific, USA‡

We discuss vacuum condensates associated with emergent QED and with torsion, as well as the possible role of the Kodama wave function in quantum cosmology.

I. INTRODUCTION

It is a great pleasure to be here and participate in this meeting. The subject matter and the SME (Standard Model Extension) approach pioneered by our host Alan Kostelecky' has been of great interest to me for a long time. In this talk, I will skim through a few specific ideas which are on my mind, and in general eschew the grand overview.[1] However, I will first define some ground rules that I shall use. They greatly overlap with Alan's ground rules, so these introductory remarks should also serve as a very brief review of Alan's program. The problem addressed in this meeting is the Big Problem: what happens to the standard model and Einstein gravity when the energy scale approaches and exceeds the Planck scale? I see this problem as quite analogous to the one which was faced in the 1950's and 1960's: what happens when the energy scale exceeds the natural scale of the strong interactions? In hindsight, and with some liberties taken with history, the low energy theory could be described in terms of an effective action. The ingredients of this effective action consisted of self-coupled pions and nucleons, with an apparent isospin symmetry and a slightly broken chiral symmetry, responsible for the relatively small mass of the pion relative to the natural strong-interaction scale. On the high-energy, short-distance side of the strong interaction barrier, the appropriate degrees of freedom turned out to be quarks and gluons, with the original degrees of freedom reinterpreted as non-fundamental, composite fields. For the Big Problem, I presume that the fate of the pion is not so different than what the fate of the photon, the gluons, the electroweak gauge bosons, and the graviton will be–they become described in terms of more fundamental degrees of freedom. It is quite probable that new concepts, as new as color and quarks were for the strong interactions, will be required–especially regarding the structure of spacetime itself. The challenge is formidable, because experiments will be much scarcer than they were in breaking through the strong-interaction barrier. But we should never give up.

II. EMERGENT QED AND GRAVITY

The idea that the photon might be a Goldstone boson of a theory of spontaneously broken Lorentz covariance goes back a long way. I myself made a try,[2] copying closely the Nambu-Jona-Lasinio formalism[3] for the Goldstone pion. This idea is better expressed nowadays in terms of effective field theory language, where one can easily presume that the Maxwell Lagrangian one wants for QED is readily obtained via loop-diagram radiative effects. I prefer to focus on this effective Lagrangian as much as possible, setting aside the detailed mechanism of how it arises (at least for now) as much as possible. So the interesting problem for me is what other terms in the effective Lagrangian come along with the ride–there may be gauge and/or Lorentz violating terms as well. The general attack on this question is beautifully expressed in terms of Alan's Standard Model Extension (SME). I personally opt to dumb down the problem by asking which of the violating terms, not present in textbook QED, might be the most important. My general rule is that gauge-variant terms should be suppressed, probably by very small coefficients, and that Lorentz-violating terms are even more suppressed. This is, because of the SME constraints, a matter of simple survival.

A few years ago, I revisited the subject,[4] and guessed that the leading gauge-variant term might be a Mexican hat potential, with a huge, GUT-scale vacuum expectation value M for the gauge potential. I chose the quartic coupling constant to be extremely small, of order 10^{-30}, in such a way that it would vanish in the limit of vanishing dark

---

* Talk presented at the Fifth Meeting on CPT and Lorentz Symmetry, University of Indiana, June 28-July 2, 2010.
† This work was supported in part by Contract No. DE AC02-76F00515.
‡Electronic address: bjorken@slac.stanford.edu

energy.

$$L = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) - \frac{\mu}{M}(\phi^2 - \mathbf{A}^2 - M^2) - (\rho\,\phi - \mathbf{J}\cdot\mathbf{A}) \tag{1}$$

Following the procedure used in the effective theory of Goldstone pions, i.e. eliminating the sigma meson in terms of the three pion fields, one can eliminate the electrostatic potential ϕ in terms of the vector potential $\mathbf{A}$, remaining at the bottom of the Mexican hat potential well. Implementing this leaves behind only the Maxwell term in the Lagrangian, but with a nonlinear relation between electric field $\mathbf{E}$ and gauge potential $\mathbf{A}$.

$$\phi = \sqrt{\mathbf{A}^2 + M^2} = M + \frac{\mathbf{A}^2}{2M} + \cdots \tag{2}$$

$$\mathbf{E} = -\dot{\mathbf{A}} - \nabla\left(\frac{\mathbf{A}^2}{2M}\right) + \cdots \tag{3}$$

When the dust settles, at tree level the effect of adding the extra Mexican-hat term amounted to fixing the gauge– although the gauge one gets is a curiously nonlinear one. The Gauss-law Maxwell equation of constraint (in the presence of an external conserved charge-density ρ becomes

$$\frac{\partial\Gamma}{\partial t} = \nabla\cdot(\mathbf{v}\Gamma) \tag{4}$$

$$\Gamma = \nabla\cdot\mathbf{E} - \rho \qquad \mathbf{v} = \frac{\mathbf{A}}{\sqrt{\mathbf{A}^2 + M^2}} \tag{5}$$

If gauge invariance is broken, as above, then there is a preferred gauge, in terms of which the theory most closely follows the dynamics of the underlying, hidden degrees of freedom. In general, it makes sense to guess the "most probable gauge". My choice is temporal gauge, and the above scenario is a specific way to express this choice. In temporal gauge, the longitudinal-photon degrees of freedom are, in a sense, dynamical, because they have non-vanishing canonical momenta. However, in practice the Gauss-law constraint makes these degrees of freedom act like a Bose condensate, described by only a few classical degrees of freedom. It is interesting that, in CPT2001, Nambu discussed just this point in his talk,[5] and ascribed this idea to work of Dirac[6] in the 1950's.

To me it would be very interesting if somehow this longitudinal-photon condensate might somehow be activated. So recently I gave it a try. The game is to stay with the Mexican-hat picture above, but to assume that the vacuum gauge-potential condensate has spacetime dependence. A very simple, cosmological type of behavior is to assume

$$F_{\mu\nu} = 0 \qquad A_\mu = \partial_\mu\,\Lambda(r,t) \tag{6}$$

Our previous example set $\Lambda = M\,t$. If we choose instead

$$\Lambda = M\,\tau \qquad \tau^2 = t^2 - \frac{r^2}{c^2} \tag{7}$$

it follows that

$$\phi = \frac{Mt}{\tau} \qquad \mathbf{A} = -\frac{M\mathbf{r}}{c^2\tau} \qquad \phi^2 - c^2\mathbf{A}^2 = M^2 \tag{8}$$

This can be constructed from the same Mexican-hat potential as before, provided that $c = 1$. I put in the Lorentz violation mostly (but not entirely) for fun, because the solution admits so easily the generalization.

The net result of this construction is a vacuum which will become, or which has been, unstable, depending upon whether we live in the past or future "lightcone" associated with the gauge function Λ. It seems to me that this might be a mechanism for catalyzing the cosmological "reheating" transition, because the onset of the instability outraces even the accelerated expansion of the universe.

What about emergent gravitons? The idea goes back to Sakharov[7],and the Einstein-Hilbert action is arguably easy to obtain via radiative loops. Again the problem is what else, if anything, comes along for the ride. A general attack can quickly lead to quite a mess.[8] At the opposite extreme, I might guess that the most important violating term is a potential $V(\sqrt{g})$, depending only upon the determinant g of the metric. The Einstein equations are than easy to obtain, and they will make trouble unless V' vanishes. This leads to a fixed value of the determinant, and a consequent "emergent unimodular gravity". So at this level I only see gauge fixing as the output consequence. Quite a lot more can be said about this approach,[9] but Alan is better equipped than I to say it[10].

III. TORSION AND CP VIOLATION

If one wishes to synthesize the standard model with gravity it would seem that the quark and lepton degrees of freedom have to be a rather central part of the story. This means that the coupling of gravity to spinors is a very relevant issue. The standard metric formalism is ill-suited to this task and one should use the first-order Einstein-Cartan (Palatini) formalism instead. The ten degrees of freedom describing the metric tensor are replaced by forty. Twenty-four are connection variables ω. They are essentially gauge potentials, a set of six four-vectors in spacetime, which live in the adjoint representation of an "internal" O(3,1) gauge group. The other sixteen degrees of freedom are the tetrad variables e, which are a set of four spacetime four-vectors living in the vector representation of O(3,1). The Yang-Mills curvature of the connection is called R. There are evidently thirty-six components of that beast. And the usual metric tensor g is defined in terms of a quadratic form built from the tetrad; the O(3,1) indices are contracted via a Minkowski metric.

In this formalism there are six terms which are in a sense "leading", at least in descriptions of gravity commonplace in the loop-gravity community. Three of them are present in ordinary metric gravity. They are the Einstein-Hilbert term, the cosmological-constant term, and the Gauss-Bonnet (Euler) topological term:

$$\begin{aligned} L_{EH} &= e^A \wedge e^B \wedge R^{CD} \wedge \epsilon_{ABCD} \\ L_{CC} &= e^A \wedge e^B \wedge e^C \wedge e^D \epsilon_{ABCD} \\ L_{GB} &= R^{AB} \wedge R^{CD} \epsilon_{ABCD} \end{aligned} \tag{9}$$

(These indices $ABCD$ live in the O(3,1) internal space; the wedge products act on spacetime indices.)

Variation of such an action with respect to the connections ω yields a constraint on the connection, namely that it be the usual Levi-Civita connection described by the Christoffel symbols. Variation with respect to the tetrad variables yields the Einstein equations. The Gauss-Bonnet/Euler term is the divergence of a topological current and does not contribute to the equations of motion. But it will reappear later in this story.

In this formalism, the connection does not start out ab initio as Levi-Civita, as in metric gravity. The difference between a general connection and the Levi-Civita choice is described by what is called torsion. While it is very arguable that torsion can be neglected in the classical theory, it is much less obvious that it can at the quantum level, especially when spinor degrees of freedom are included. If torsion is admitted, it is also more natural to admit additional terms into the gravitational action, in particular terms which are CP odd. It turns out that there are three "leading terms" that can most naturally be included. The most interesting one is called the Holst term,[11] which is a close analog of the Einstein-Hilbert term. The other two, the Pontryagin and Nieh-Yan terms,[12] are both topological. Like the Gauss-Bonnet term, they do not affect equations of motion. But they might lead to subtle quantum effects similar to what happens with the (Pontryagin) topological term $\mathbf{E} \cdot \mathbf{B}$ in QCD:

$$\begin{aligned} L_H &= e^A \wedge e^B \wedge R_{AB} \\ L_P &= R^{AB} \wedge R_{AB} \\ L_{NY} + L_H &= (De)^A \wedge (De)_A \end{aligned} \tag{10}$$

(Here D is the covariant derivative with respect to the connection ω; see Equation 13 below for some details.)

I find it impressive that this formalism for pure gravity so naturally admits CP violation. And from an effective-field-theory point of view, there is no reason to omit such terms ab initio, because we know that CP violation occurs in nature.

It is especially important to understand the role of the Holst term, which is not pure topological. The coefficient of the Holst term in the action is inversely proportional to what is known as[13] the Barbero-Immirzi parameter γ. In the first order formalism, especially as elucidated by the loop-gravity community, this Holst term creates mixing between torsion degrees of freedom and ordinary metric degrees of freedom, but in such a way that the Einstein equations for ordinary macroscopic applications remain unaffected. But the formal canonical theory, used to set up quantization, is deeply affected.[14]

IV. AN AXIAL-VECTOR CONDENSATE

Just as I did for the Goldstone photon, I have tried to "activate" such torsion degrees of freedom in as simple a way as possible, in order to see how they might enter into phenomenology. This led to presuming that there might exist, for some fermionic degrees of freedom (either standard-model or beyond-the-standard-model), a Lorentz- violating vacuum condensate of axial vector current:

$$< \overline{\Psi} \gamma_5 \gamma_\mu \Psi > = \ \eta_\mu \rho_A \tag{11}$$

Here η_μ is a unit timelike vector, at rest in the CMB rest frame. The torsion part of the connection appears in the Dirac action. In first order form the Dirac operator is rewritten as follows:

$$\gamma^\mu \nabla_\mu \longrightarrow \epsilon_{ABCD}(e^A \wedge e^B \wedge e^C \gamma^D) \wedge D \tag{12}$$

The covariant derivative is

$$D_\mu = \partial_\mu + \frac{1}{2}\omega_\mu^{EF}\gamma_E\gamma_F \tag{13}$$

The condensate therefore leads to a source term which couples linearly to the connection and in particular to a piece of the torsion known in the trade as contorsion. In the case of ΛCDM cosmology the mathematics is simple enough for even me to do. The nonvanishing components of the tetrad and the connection are

$$\begin{aligned} e_t^0 &= N \\ e_x^1 = e_y^2 = e_z^3 &= a(t) \\ \omega_x^{01} = \omega_y^{02} = \omega_z^{03} &= K(t) \\ \omega_x^{23} = \omega_y^{31} = \omega_z^{12} &= C(t) \end{aligned} \tag{14}$$

N is what is known in the trade as lapse, a constant which determines the rate of ticking of the FRW clock. It can be set to unity after the equations of motion have been obtained from the variational principle. $a(t)$ is the familiar FRW scale factor. The quantity $K(t)$ is known in the trade as extrinsic curvature, while $C(t)$ measures the contorsion.

With this starting point, it is not too hard to explicitly construct equations of motion, solve them, and connect this language to the usual textbook metric-gravity language. When the dust settles, one finds that there is a dark-energy-like contorsion given by

$$C(t) = \frac{2\pi a(t)\gamma^2\rho_A}{M_{\rm pl}^2(1+\gamma^2)} \tag{15}$$

(γ is the Barbero-Immirzi parameter.)

There is an important combination $A(t)$ of extrinsic curvature and contorsion

$$A(t) = K(t) + \frac{C(t)}{\gamma} \tag{16}$$

The Einstein equations, expressed in terms of A, are essentially unaffected by the presence of the nonvanishing contortion C. Its only effect is to renormalize the dark energy scale. Let H be the asymptotic, dark-energy-dominated, expansion rate, as given by $a(t) = e^{Ht}$. This quantity depends upon C, such that

$$H^2 = H_{\rm cc}^2 - \frac{(4\pi\gamma\rho_A)^2}{M_{\rm pl}^4(1+\gamma^2)}. \tag{17}$$

Here $H_{\rm cc}$ is evidently the value the Hubble parameter would take in the absence of torsion and the axial condensate. If this renormalization of the dark energy scale is of order unity, one has

$$\frac{4\pi\gamma\rho_A}{\sqrt{1+\gamma^2}} \;\sim\; HM_{\rm pl}^2 \;\sim\; 10^{-60}M_{\rm pl}^3 \;\sim\; (10^{-20}M_{\rm pl})^3 \;\sim\; \Lambda_{QCD}^3 \tag{18}$$

This is what I call the Zeldovich relation: in natural units the cube of the QCD scale is of order the Hubble scale. It was noticed by Zeldovich[15] in 1967 and has been occasionally been rediscovered in the interim.[16] I encounter it often in my speculative excursions into trying to understand the dark energy problem, and I now take it seriously. I find that this is a minority viewpoint. Most people seem to dismiss the Zeldovich relation as a numerical coincidence.

This axial condensate has another consequence. Because all spinor degrees of freedom couple to gravity, they must all feel the effect of the vacuum contorsion. This leads to a Lorentz-violating term in the effective action, one which is prominent in the SME catalog.[17]

$$L' = b_\mu \overline{\psi}\gamma^\mu\gamma_5\psi \tag{19}$$

The condensate contribution to this Kostelecky' b - parameter is

$$b_\mu = \eta_\mu \frac{2\pi\gamma^2\rho_A}{M_{\rm pl}^2(1+\gamma^2)} \tag{20}$$

Here η_μ is a unit timelike vector, pure timelike in the CMB rest frame. If the Zeldovich relation holds, then

$$b_\mu \leq 10^{-33} eV \tag{21}$$

The effect is a billion times smaller than the experimental limit, unless the condensate density is taken to be much higher than its "natural" value. Such behavior would be appropriate for scenarios in which there is a fine-tuned cancellation of the torsion contribution with a much larger "primordial" dark energy. I think it is well worth some effort to push the experimental limits on b if it is not too difficult to do so.

At this meeting I learned of closely related work of Poplawski.[18] He uses the QCD quark vacuum condensates instead of a Lorentz-violating axial condensate to arrive at a very similar endpoint. This very interesting work is evidently much more conservative in nature than my utilization of a Lorentz-violating condensate; hence it is more credible.

V. VACUUM PHASE DENSITY

The last topic in this potpourri is one in which the Zeldovich relation again appears. Consider a finite box of spatially flat FRW ΛCDM universe, with periodic boundary conditions[19] applied ("compactification on a torus"). As time goes on, this box will expand. The dimensions of the box are controlled by the FRW scale factor, which evolves according to the Einstein equations of cosmology. If the box contains only pure dark energy, it will expand exponentially, with a doubling time for its volume of about 3 1/2 billion years. As far as I am concerned, all one needs at initial times is a liter of the stuff; the problem of what is going on at the microscopic level within such a box is the fundamental problem of dark energy. The semiclassical wave function of this box of dark energy is the exponential of a phase factor, given by the classical action. It turns out to be proportional to the volume of the box. The coefficient of this phase factor is linear in the Hubble parameter, in natural units. This leads to the conclusion that the characteristic volume, for which the "phase density" is of order 2π, is of order the QCD scale-the Zeldovich relation again applies.

There is an interesting subplot to this story, which originates in a variant of first-order gravity invented[20] by MacDowell and Mansouri and elaborated recently by Freidel and Starodubtsev.[21] The idea is to synthesize the tetrad and connection variables (e, ω) of the first order theory into a single grand connection A which lives in an internal O(4,1) space. There are $4 \times 10 = 16 + 24$ slots to fill for such a connection A, just the right number:

$$A^{5A}_\mu = H_{\rm cc}\; e^A_\mu \qquad A^{AB}_\mu = \omega^{AB}_\mu \qquad (A, B = 0, 1, 2, 3) \tag{22}$$

A 6 x 10 field strength (curvature) F can be defined in the usual way from the connection A. In terms of this F, the Einstein-Hilbert action, complete with cosmological term and Euler (Gauss-Bonnet) term, takes a very simple form. First contract the gauge potential (and field strength) into gamma matrices:

$$A_\mu \longrightarrow \frac{1}{2} A^{AB}_\mu \gamma_A \gamma_B \tag{23}$$

Then the Lagrangian density is simply

$$L = \frac{M^2_{\rm pl}}{H^2_{\rm cc}} \,{\rm Tr}\, \gamma_5 F \wedge F \tag{24}$$

Noteworthy is the large dimensionless coefficient of the action, of order 10^{120}. To get the additional three CP violating terms defined in Eqn. 10, one simply makes the replacement $F \to e^{\gamma_5 \theta} F$ in the above equation; the (constant) Barbero-Immirzi parameter γ is then given by $\cot\theta$.

This way of expressing gravity is provocative and certainly invites its use as a starting point for enlarging the theory in some way to encompass standard model degrees of freedom.[22] However, that is not the issue here. Instead it is easy to find that imposing a "gauge condition" $F = 0$ for the field strength associated with the connection A leads to nontrivial solutions. In particular, deSitter space, which describes our expanding box of dark energy, is such a solution. According to this interpretation, the vacuum phase density, given by the exponential of the MacDowell-Mansouri action (which is quadratic in the field strength F) should vanish.

The resolution of this paradox is that the Gauss-Bonnet term, although pure topological, does contribute vacuum phase. And the MacDowell-Mansouri construction guarantees that this topological contribution to the phase density cancels out the contribution given by the standard metric theory. This Gauss-Bonnet term, complete with a remarkably large coefficient of 10^{120}, is essentially what is known in the loop-gravity community as the Kodama wave function.[23] However, there it plays a different–and controversial–role.[24]

In any case, what is suggested here is that vacuum topology might be an important ingredient in the understanding of the Zeldovich relation, of course assuming–as I always do–that it is more than a numerical accident.

VI. SUMMARY

I see the main messages in the above discussion to be as follows:

1. If the "Planck Barrier" is analogous to the "Confinement Barrier" faced by the physics of the 1950's and 1060's, the techniques used successfully then may be relevant again. These include emphasis on local currents and their operator-product algebra. There are a huge number of two-point functions to consider in this regard, as well as a huge number of anomaly equations. I do not think they have been fully investigated. Topology, especially vacuum topology, probably should be in foreground. Of course there may be new gauge degrees of freedom to be identified (analogous to color for the strong interactions)–this is an old story. And it should be recognized that the role of flavor symmetry, put front and center in the old days, turned out to be a misleading one. Isospin is now regarded as an accidental symmetry, and flavor has yet to submit to a gauge principle. Maybe that will be true again. Sometimes I like to envisage a flavorless toy universe with only the third generation included. It is much simpler and cleaner than real life, so much so that I am occasionally tempted to regard it as a realistic template for the full theory, to be later embellished by first- and second-generation infrared decorations.

2. In order to synthesize the standard model degrees of freedom with gravity, the first order Palatini formalism should be embraced. It admits torsion and CP violation in a natural way, and this difference may well end up being real physics, not mere formalism. Furthermore, the MacDowell-Mansouri / Freidel-Starodubtsev extension in a sense makes the theory "almost topological", and this viewpoint may also be a novel one relative to the standard metric language.

3. If gauge invariance is emergent, then there is a preferred choice of gauge, in terms of which the physics of the underlying theory is best expressed. And those gauge degrees of freedom, while largely hidden from view because of the strong SME experimental constraints, might still be "activated" at some level. I would guess that the effects of such activation vanish in the limit of vanishing dark energy. This hypothesis puts the setting of this problem at only the deepest level, as well as providing some protection from the often severe SME experimental limits.

4. The cosmological, QCD, and electroweak vacuua, which are all too easy to compartmentalize because of the disparate distance scales associated with each, deserve to be synthesized. They occupy the same region of spacetime, have the same energy, and are coupled dynamically to each other. The Zeldovich relation may be evidence that in fact they leave nontrivial imprints on each other and are better treated as a single entity.

5. The relatively prominent role of the Zeldovich relation argues for the vacuum structure of QCD to play a central role in shaping overall vacuum structure. Therefore there is a motivation to identify candidate "vacuum condensates" specifically associated with QCD. I like to entertain the notion that there is a condensate of topological charge N (the quantity canonically conjugate to the θ parameter of QCD CP violation). The total mean N in a Hubble-scale universe is then of order 10^{120} instead of the usually-assumed value of zero. The uncertainty in N from instanton-induced transitions is then of order 10^{80} or so instead of the usually-assumed value of infinity. And Zhitnitsky[25] and his collaborators recently have also argued for a QCD-related origin of dark energy. Their condensate does involve vacuum topology, and does connect with the Zeldovich relation. However, they choose a covariant gauge, and "activate" the "Veneziano-ghost" degrees of freedom associated with the topological susceptibility. I find all this very interesting, but would prefer seeing their arguments recast in a more physical gauge (e.g. temporal gauge) in the light of my comments in item 3 above.

VII. ACKNOWLEDGEMENTS

Thanks go to my colleagues at SLAC and Stanford for patient criticism of these speculations, and in particular to Ronald Adler, Jeffrey Scargle, Alex Silbergleit, Robert Wagoner, and Marvin Weinstein. I also thank Alan Kostelecky' for organizing this superb meeting, and to Nikodem Poplawski for conversations regarding the work in reference 18.

[1] For a more general personal overview, see "The Future and its Alternatives", www.slac.stanford.edu/grp/th/symposia.
[2] J. Bjorken, Ann. Phys. **24**, 174 (1963)
[3] Y. Nambu and G. Jona-Lasinio,Phys. Rev. **122**, 345 (1961); **124**, 246 (1961)
[4] J. Bjorken, arXiv:0111196.

[5] Y. Nambu, Second Meeting on CPT and Lorentz Symmetry, ed. A. Kostelecky', World Scientific (2001), p. 3.
[6] P. Dirac, Proc. Roy. Soc. **A209**, 291 (1951).
[7] A. Sakharov, Gen. Rel. Grav. **32**, 365 (2000); translation from Dokl. Akad. Nauk. SSSR **170**, 70 (1967).
[8] P. Kraus and E. Tomboulis, arXiv:0203221
[9] M. Henneaux and C. Teitelboim, Phys. Lett. **B222**, 195 (1989)
[10] A. Kostelecky' and R. Potting, arXiv:0901.0662, and references therein.
[11] S. Holst, arXiv:0901.0662
[12] O. Chandia and J. Zannelli, arXiv:9702025 and arXiv:9708138; H. Nieh and C.N. Yang, Int. J. Mod. Phys. **A22**, 5237 (2007), and references therein.
[13] F. Barbero, Phys. Rev. **D51**, 5498 and 5507 (1995); G. Immirzi, arXiv:9701052
[14] M. Bojowald and R. Das, Phys. Rev. **D78**, 064009 (2008); arXiv:0710.5722.
[15] Ya. Zeldovich, JETP Lett. **6**, 316 (1967).
[16] For example, see S. Carniero, arXiv:0305081; R. Schutzhold, arXiv:0204018; A. Randono, arXiv:0805.2955; see also references 18 and 25.
[17] A. Kostelecky', N. Russell, and J. Tasson, arXiv:0712.4393
[18] N. Poplawski, arXiv:1005.0893; see also A. Randono, arXiv:1005.1294.
[19] J. Levin, Phys. Rept. **365**, 251 (2002); arXiv:0108043.
[20] S. MacDowell and F. Mansouri, Phys. Rev Lett. **38**, 739 and 1376 (1977).
[21] L. Freidel and A. Starodubtsev, arXiv:0501191
[22] See, for example, G. Trayling, arXiv:9912231; G. Lisi, arXiv:0711.0770.
[23] H. Kodama, Phys. Rev. D42, 2548 (1990); L. Smolin, hep-th/0209079.
[24] E. Witten, arXiv:0306083 and references therein.
[25] F. Urban and A. Zhitnitsky, arXiv:0906.2162, arXiv:0906.2165), and arXiv:0909.2684.